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ACOUSTIC MODES IN FLUID NETWORKS

C.D. Michalopoulos
McDonnell Douglas Space Systems Co.
Houston, Texas

Robert W. Clark, Jr.
McDonnell Douglas Space Systems Co.
Houston, Texas

Harold H. Doiron
McDonnell Douglas Space Systems Co.
Houston, Texas

SUMMARY

Pressure and flow rate eigenvalue problems for one-dimensional flow of a fluid in a network of pipes are derived from the familiar transmission line equations. These equations are linearized by assuming small velocity and pressure oscillations about mean flow conditions. It is shown that the flow rate eigenvalues are the same as the pressure eigenvalues and the relationship between line pressure modes and flow rate modes is established. A volume at the end of each branch is employed which allows any combination of boundary conditions, from open to closed, to be used.

The Jacobi iterative method is used to compute undamped natural frequencies and associated pressure/flow modes. Several numerical examples are presented which include acoustic modes for the Helium Supply System of the Space Shuttle Orbiter Main Propulsion System.

It should be noted that the method presented herein can be applied to any one-dimensional acoustic system involving an arbitrary number of branches.

INTRODUCTION

Often in the analysis of dynamic responses of piped fluid networks, a preliminary "quick look" at acoustic mode shapes and frequencies of the system is a useful diagnostic tool prior to the initiation of more detailed diagnostic testing or modeling efforts. Knowledge of the fundamental and higher order response mode frequencies of the system based on linear analysis allows for rapid assessment of modes which may couple dynamically with devices such as regulators and check valves. This provides valuable diagnostic information when troubleshooting dynamic problems with these types of devices. Knowledge of pressure and flow mode shapes can provide guidance on positioning of high frequency pressure and flow transducers during testing. Such information can be used to infer magnitude of pressure and flow oscillations in regions of the fluid system where measurements cannot be made due to various practical limitations typically encountered on operational systems.

During the course of diagnostic studies of several dynamic phenomena with regulators, check valves, propellant feed systems and rocket engines of the Space Shuttle Orbiter

spacecraft over the last four years, the authors developed a systematic fluid element approach to the analysis of related piped fluid networks. These modeling procedures were incorporated into a FORTRAN computer program called ACLMODES.

This paper presents derivations of basic building block equations used in the program, illustrates numerical accuracy of the computer code on several problems with known closed-form solutions and illustrates how the program was used to analyze several dynamic phenomena associated with piped fluid networks of the Orbiter spacecraft.

GOVERNING EQUATIONS

The basic equations employed in this paper are the familiar pneumatic/hydraulic transmission line equations which govern one-dimensional transient flow. These equations for an unbranched acoustic line are first re-cast in matrix form. Then, this matrix formulation is generalized for systems of branched acoustic lines, referred to as "fluid networks".

Matrix Form of Equations for Unbranched Acoustic Lines

The ordinary differential equations governing one-dimensional flow of an ideal gas, in terms of volumetric flow, are (see Figure 1 for notation)

$$I_i \dot{Q}_i = P_i - P_{i+1} - R_{fi} |Q_i| Q_i \tag{1}$$

$$C_i \dot{P}_i = Q_{i-1} - Q_i \; ; \qquad i = 1, N \tag{2}$$

where:

I_i = inertance of the i^{th} fluid element,
C_i = capacitance of the i^{th} fluid element,
Q_i = mass flow into the i+1 element,
P_i = pressure at the center of the i^{th} fluid element
R_{fi} = resistance,
N = total number of fluid elements used to model a line segment

For a uniform line modeled with equal-length elements, the inertance, capacitance, and flow resistance are the same for all elements and are given by:

$$I = \frac{\rho L}{A} \tag{3}$$

$$C = \frac{AL}{\gamma p RT} \tag{4}$$

$$R_f = \frac{\rho}{2A^2} f\left(\frac{L + L_e}{D}\right) \tag{5}$$

where:

L = fluid element length
A = flow area
γ = polytropic process exponent
T = temperature
R = gas constant
ρ = density
f = friction factor (pipe flow)
D = line internal diameter
L_e = equivalent length for minor losses

It should be noted that Eq. (1) may be easily derived by integrating once the one-dimensional momentum equation and neglecting the convective terms. Equation (2) is the equation of conservation of mass for isentropic flow of an ideal gas. These equations are derived in References [1], [2], and [3].

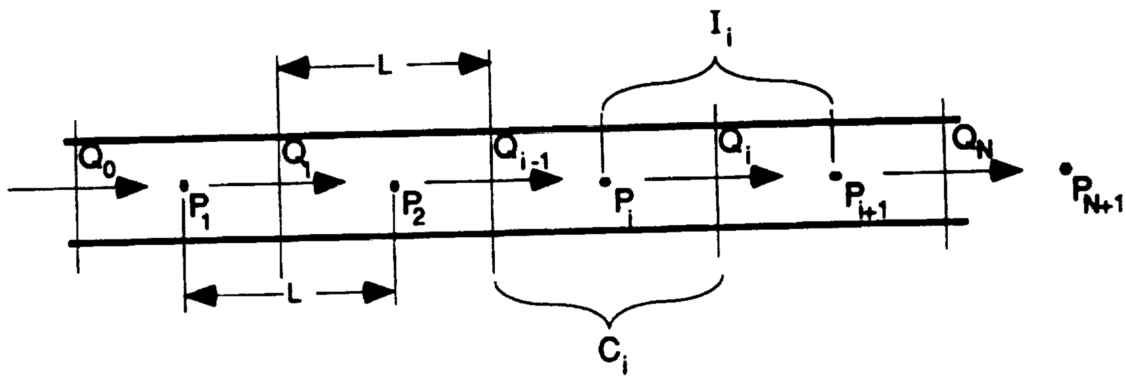

Figure 1. Typical discretization of a line segment.

The sets of Eqs. (1) and (2) may be written in matrix form in the special case of $R_f \equiv 0$, that is, for the undamped system. The matrix equations are

$$\mathbf{A}\,\dot{\mathbf{Q}} + \mathbf{B}\,\mathbf{P} = \mathbf{F}_1 \tag{6}$$

$$\mathbf{C}\,\dot{\mathbf{P}} - \mathbf{B}^T\,\mathbf{Q} = \mathbf{F}_2 \tag{7}$$

where the superscript denotes the transpose of the matrix, and:

$$\mathbf{A} = \begin{bmatrix} I_1 & & & O \\ & I_2 & & \\ & & \ddots & \\ O & & & I_N \end{bmatrix} \tag{8}$$

$$\mathbf{C} = \begin{bmatrix} C_1 & & & \bigcirc \\ & C_2 & & \\ & & \ddots & \\ \bigcirc & & & C_N \end{bmatrix} \tag{9}$$

$$\mathbf{B} = \begin{bmatrix} -1 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 \\ & & \cdot & \cdot & \cdot & \cdot & \\ 0 & 0 & 0 & 0 & \cdots & -1 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -1 \end{bmatrix} \tag{10}$$

$$(B_{i\,i} = -1, \qquad B_{i\,i+1} = 1, \qquad \text{otherwise } B_{i\,j} = 0)$$

$$\mathbf{Q} = \begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{Bmatrix}; \qquad \mathbf{P} = \begin{Bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{Bmatrix} \tag{11}$$

$$\mathbf{F}_2 = \begin{Bmatrix} Q_0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{Bmatrix}; \qquad \mathbf{F}_1 = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ -P_{N+1} \end{Bmatrix} \tag{12}$$

Differential Equations for Acoustic Lines with Branches

Consider a branched acoustic system such as that shown in Figure 2. The end volumes V_1, V_2 and V_3 are used in the formulation for generalizing the boundary conditions. It should be noted (see Appendix A) that $V = 0$ represents a closed end while $V = \infty$ is an open end.

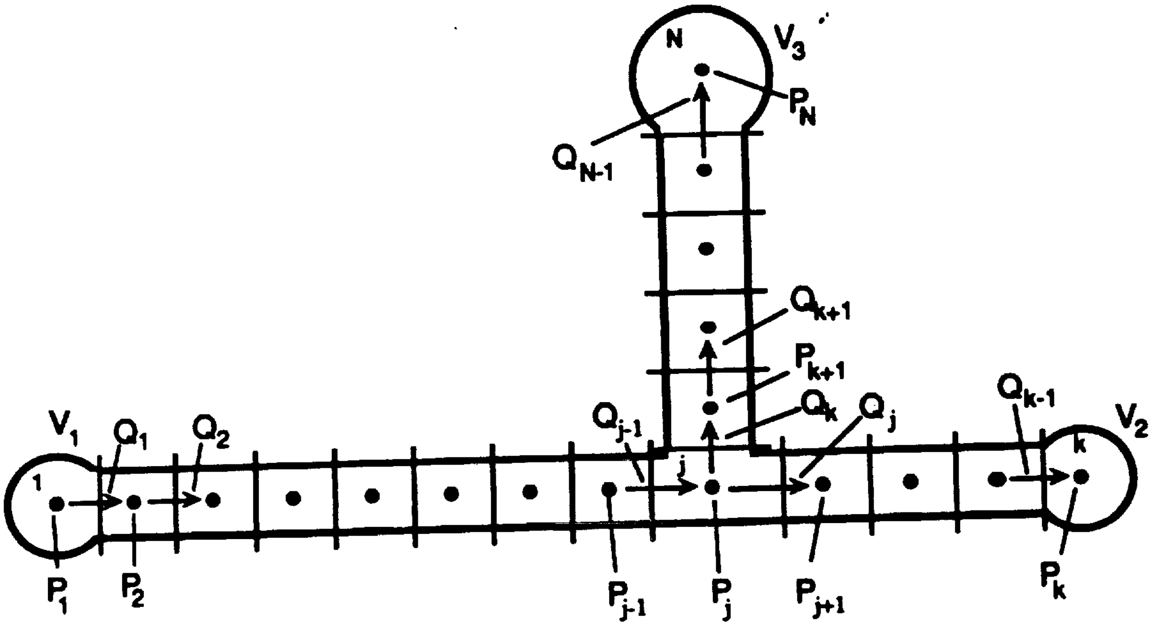

Figure 2. Example of an acoustic line with a branch.

For simplicity, damping is neglected in this section. Equations (6) and (7) are applicable in this case, but the matrix **B** has a different structure from that of Eq. (10). Note that Eqs. (1) and (2) with $R_{f_i} = 0$ apply at all elements with some modifications at the ends (elements 1, K, and N) and element j, where the branch connects to the main line. The first-order equations for these special elements are:

$$C_1 \dot{P}_1 = Q_o - Q_1 \tag{13}$$

$$C_j \dot{P}_j = Q_{j-1} - Q_j - Q_k \tag{14}$$

$$C_k \dot{P}_k = Q_{k-1} \tag{15}$$

$$C_N \dot{P}_N = Q_{N-1} \tag{16}$$

$$I_k \dot{Q}_k = P_j - P_{K+1} \tag{17}$$

with $C_1 = \dfrac{V_1}{\gamma RT}$. The **B** matrix in this case (one branch) has the following structure:

$$\begin{aligned} B_{i\,i} &= -1; \quad i = 1, N; \quad i \neq k \\ B_{k\,j} &= -1 \\ B_{i\,i+1} &= 1; \quad i = 1, N \end{aligned} \tag{18}$$

Note that **B** is an N x (N + 1) rectangular matrix.

An example on the structure of **B**:

$$j = 2$$
$$k = 4$$
$$N = 7$$

$$(7 \times 8)$$

$$\mathbf{B} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \tag{19}$$

The extension to acoustic lines with multiple branches, or fluid networks, is straight forward. Obviously, the structure of **B** depends on the numbering system used.

Linearized Form of Governing Equations

For small pressure/flow oscillations, it can be shown that

$$\mathbf{A}\,\ddot{\mathbf{q}} + \mathbf{D}\,\dot{\mathbf{q}} + \mathbf{E}_Q\,\mathbf{q} = \mathbf{g}_1 \tag{20}$$

$$\mathbf{C}\,\ddot{\mathbf{p}} + \mathbf{H}\,\dot{\mathbf{p}} + \mathbf{E}_p\,\mathbf{p} = \mathbf{g}_2 \tag{21}$$

where **A** and **C** are given by Eqs. (8) and (9) respectively. The diagonal damping matrix **D** is defined by

$$\mathbf{D} = \begin{bmatrix} \beta_1 & & & O \\ & \beta_2 & & \\ & & \ddots & \\ O & & & \beta_N \end{bmatrix}$$

where $\beta_i = 2R_{fi}Q_i^o$ is the linear damping coefficient, Q_i^o being the mean (steady) flow rate.

Note that the vectors **q**, **p**, $\mathbf{f}_1$ and $\mathbf{f}_2$ are defined according to Eqs. (11) and (12), with **Q**, **P**, $\mathbf{F}_1$ and $\mathbf{F}_2$ replaced by **q**, **p**, $\mathbf{f}_1$ and $\mathbf{f}_2$ respectively. The matrices **H**, $\mathbf{E}_q$, $\mathbf{E}_p$, $\mathbf{g}_1$ and $\mathbf{g}_2$

in Eqs. (20) and (21) are given by

$$\mathbf{H} = \mathbf{B}^T \mathbf{A}^{-1} \mathbf{D} \mathbf{B}^{T^{-1}} \mathbf{C} \tag{22}$$

$$\mathbf{E}_Q = \mathbf{B} \mathbf{C}^{-1} \mathbf{B}^T \tag{23}$$

$$\mathbf{E}_P = \mathbf{B}^T \mathbf{A}^{-1} \mathbf{B} \tag{24}$$

$$\mathbf{g}_1 = \dot{\mathbf{f}}_1 - \mathbf{B} \mathbf{C}^{-1} \mathbf{f}_2 \tag{25}$$

$$\mathbf{g}_2 = \dot{\mathbf{f}}_2 + \mathbf{B}^T \mathbf{A}^{-1} (\mathbf{f}_1 + \mathbf{D} \mathbf{B}^{T^{-1}} \mathbf{f}_2) \tag{26}$$

ACOUSTIC MODES IN FLUID NETWORKS

The undamped natural frequencies and mode shapes for a fluid network are determined from Eqs. (20) and (21) with $\mathbf{D} = 0$. Setting the right hand of these equations equal to zero, the free, undamped flow/pressure oscillations in a fluid network are governed by

$$\mathbf{A} \ddot{\mathbf{q}} + \mathbf{E}_Q \mathbf{q} = 0 \tag{27}$$

$$\mathbf{C} \ddot{\mathbf{p}} + \mathbf{E}_P \mathbf{p} = 0 \tag{28}$$

where $\mathbf{A}$, $\mathbf{C}$, $\mathbf{E}_Q$ and $\mathbf{E}_P$ are defined by Eqs. (8), (9), (23) and (24) respectively.

The eigenvalue problem associated with Eq. (27) is

$$\mathbf{A} \mathbf{X} = \lambda_Q \mathbf{E}_Q \mathbf{X} \tag{29}$$

where $\lambda_Q = \frac{1}{\omega_Q^2}$ and $\mathbf{X}$ is a flow eigenvector or flow mode.

The eigenvalue problem associated with Eq. (28) is

$$\mathbf{C} \mathbf{Y} = \lambda_P \mathbf{E}_P \mathbf{Y} \tag{30}$$

where $\lambda_P = \frac{1}{\omega_P^2}$ and $\mathbf{Y}$ is a pressure eigenvector or pressure mode.

It can be easily shown that

$$\omega_Q = \omega_P \tag{31}$$

and

$$\mathbf{Y} = \mathbf{C}^{-1} \mathbf{B}^T \mathbf{X} \tag{32}$$

Computer Program ACLMODES

A computer program, referred to as ACLMODES, was developed which computes the natural frequencies and associated flow rate/pressure modes for an acoustic line network. The program can accommodate any number of branches with any combination of boundary conditions (ranging from closed to open at each end). The input to the program is relatively simple due to its capability of generating acoustic elements with identical properties.

The program employs the Jacobi Iterative Method to solve the eigenvalue problem defined by either Eq. (29) or Eq. (30). Line pressure modes are computed either directly from Eq. (30) or using Eq. (32).

NUMERICAL RESULTS AND DISCUSSION

Three examples demonstrating capabilities of the ACLMODES program are shown in the following paragraphs. The first example is a comparison of a simple system's frequencies predicted by ACLMODES and the closed-form solution shown in Appendix A. The other examples are actual applications of ACLMODES on Space Shuttle fluid line systems.

Numerical Test Case

Example 1 is a test case consisting of a 100 inch long pipe of 0.5 inch I.D. filled with helium and a volume on both ends. Three different combinations of end volumes, shown in Table 1, were used. Note that all volumes are in cubic inches. Case A represents an open-closed boundary conditions and Case B a closed-closed.

	VOLUME A	VOLUME B
CASE A	0.00001	10000
CASE B	0.00001	0.00001
CASE C	0.1	10

Table 1. Volume sizes for ACLMODES test case.

ACLMODES was used to determine the first three natural frequencies of each of the three cases. Each case was repeated with four different element lengths to evaluate solution accuracy versus number of line elements employed. The closed-form solution shown in Appendix A was then used to calculated the frequencies of the three cases. The ACLMODES results are shown in Table 2 together with the closed-form solution.

	MODE	NUMBER OF ELEMENTS 10	20	50	100	CLOSED FORM
CASE A	1	100.30	100.38	100.40	100.40	100.40
	2	298.23	300.31	300.98	300.98	301.00
	3	488.85	498.43	501.13	501.52	501.64
CASE B	1	199.83	200.45	200.62	200.64	200.65
	2	394.73	399.66	401.04	401.24	401.30
	3	579.92	596.40	601.06	601.73	601.95
CASE C	1	145.02	145.09	145.10	145.11	145.11
	2	320.47	322.36	322.89	322.96	322.98
	3	502.28	511.27	513.81	514.17	514.28

Table 2. Acoustic Frequencies (Hz) of Cases A, B and C.

The percent difference between the values ACLMODES predicted and that of the closed-form solution are shown in Table 3.

	MODE	NUMBER OF ELEMENTS 10	20	50	100
CASE A	1	0.10	0.02	0.00	0.00
	2	0.92	0.23	0.01	0.01
	3	2.55	0.64	0.10	0.02
CASE B	1	0.41	0.10	0.01	0.00
	2	1.64	0.41	0.06	0.01
	3	3.66	0.92	0.15	0.04
CASE C	1	0.06	0.01	0.01	0.00
	2	0.78	0.19	0.03	0.01
	3	2.33	0.59	0.09	0.02

Table 3. Percent Error of Cases A, B and C
Compared to Closed-Form Solution

These numerical results show excellent agreement with the closed-form solution results. As expected, there is improvement in accuracy of the numerical solution as the number of line elements increases. A general rule of thumb for an acceptable line element length required to obtain accurate numerical results is $\omega L/c < 0.5$, where ω is the estimated circular frequency of the mode sought in rad/sec, L is the line element length (inch) and c is the speed of sound in the fluid (inch/sec). Figure 1 shows percent error versus $\omega L/c$ for all values in Table 3.

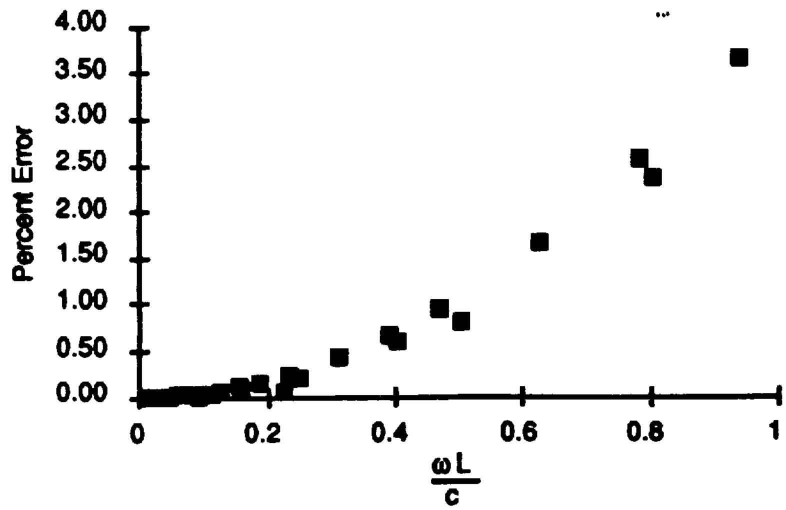

Figure 1. Percent error versus ωL/c

It can be seen that when the condition $\omega L/c < 0.5$ is not met the percent error is greater than 1.

Test Stand Line Dynamics

Stability testing of the Primary Reaction Control System (PRCS) thruster at the NASA White Sands Test Facility (WSTF) required that the test stand have similar line dynamics to that of the Space Shuttle Orbiter. This is because the PRCS thruster is a pressure-fed engine so the pressure recovery or waterhammer of the line governs the start-up transients. Therefore, a simple line having similar waterhammer characteristics to that of the aft PRCS feed system, which is a fairly complicated system with many branches and twelve primary thrusters (see Figure 2), was desired.

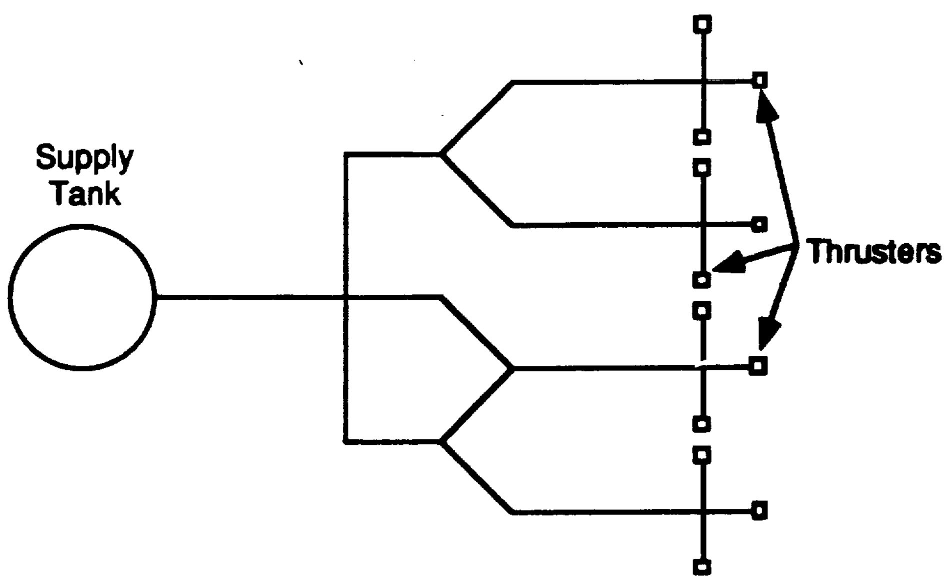

Figure 2. Schematic of the Space Shuttle's PRCS aft fuel supply system.

The original idea was to use similar line diameters to that of the vehicle and the average distance from the supply tank to the thrusters as the test stand line length. However, it was not obvious that this would yield the same dynamics as the vehicle so a model of each configuration was constructed.

The first mode of the proposed test stand fuel line was found using ACLMODES and is shown in Figure 3. As expected it appears to be an open-closed mode with a frequency of 65 Hz. The first mode of the vehicle's piping system was found to have a frequency of 40 Hz and is shown in Figure 4. The difference in frequency was not acceptable so the test stand line was reconfigured to have the same first natural frequency as the vehicle feed system.

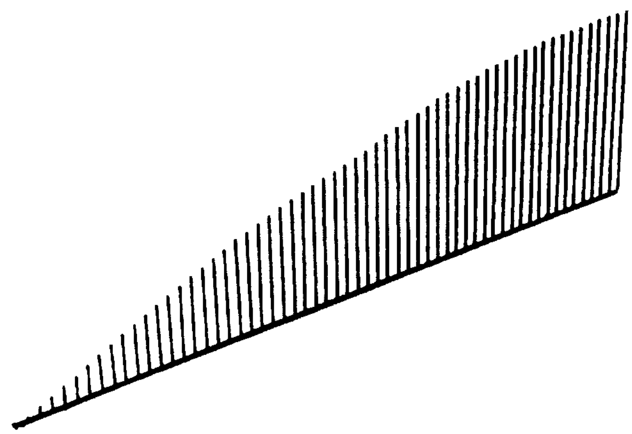

Figure 3. First pressure mode of simple feed system.

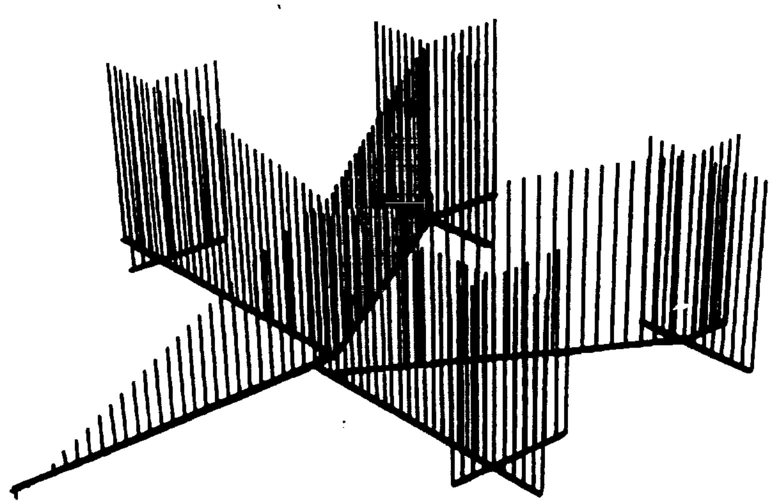

Figure 4. First pressure mode of the Space Shuttle's PRCS aft fuel supply system.

Space Shuttle Main Propulsion System Helium Supply System

The regulators in the helium supply system for Space Shuttle Main Propulsion System (MPS) were experiencing oscillations. These oscillations were seen both on test stands as well as on the vehicle. It was believed that the source of the oscillations was the regulators coupling with the downstream line acoustics.

Each engine has its own helium supply system. A helium supply system consists of high pressure supply tanks, tubing leading up to two panels in parallel, and lines from the panels rejoining and continuing on to the engine. A panel consist of a regulator and relief valve as well as several solenoids and check valves. Only the lines downstream of the regulators were of interest, so they were all that was modeled (see Figure 5).

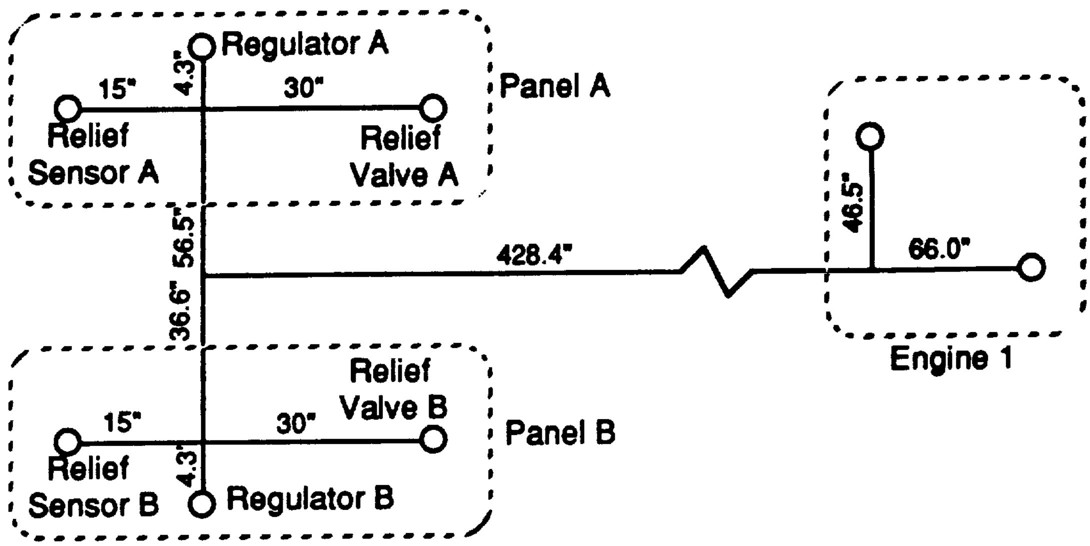

Figure 5. MPS Engine 1 helium schematic of lines downstream of regulators.

Figure 6 through 8 show the first, fourth and eighteen pressure modes predicted by ACLMODES for the Engine 1 helium supply system. The fourth mode is shown again in Figure 9 as a flow mode.

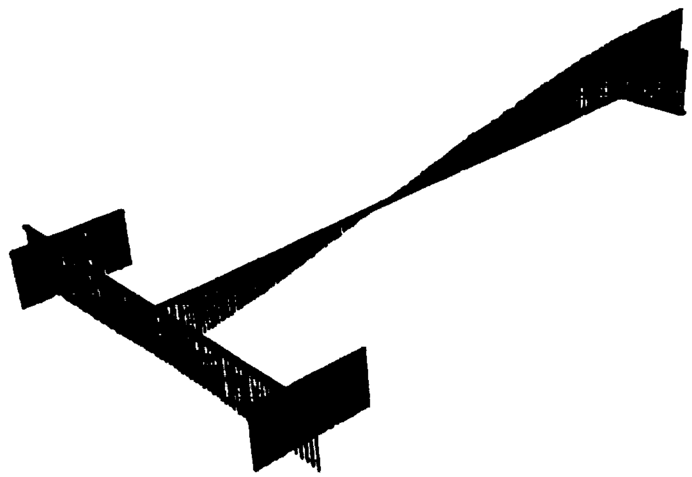

Figure 6. First pressure mode of the Engine 1 helium lines downstream of regulators.

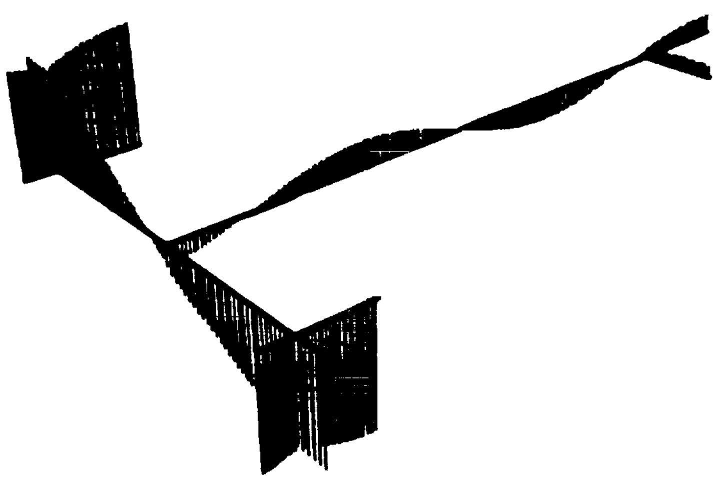

Figure 7. Fourth pressure mode of the Engine 1 helium lines downstream of regulators.

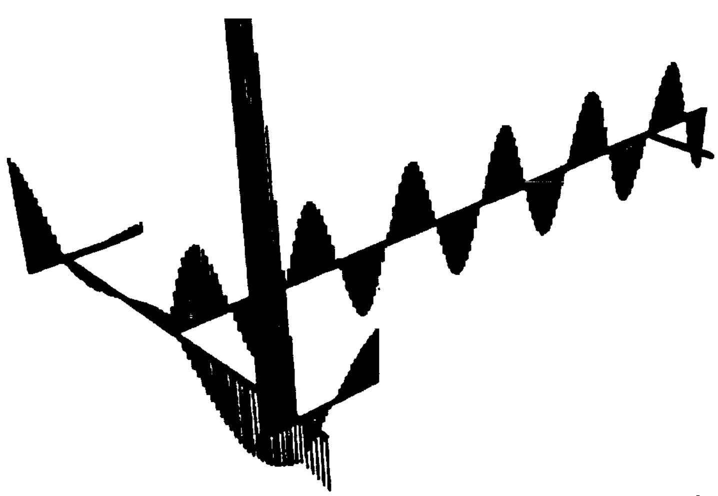

Figure 8. Eighteenth pressure mode of the Engine 1 helium lines downstream of regulators.

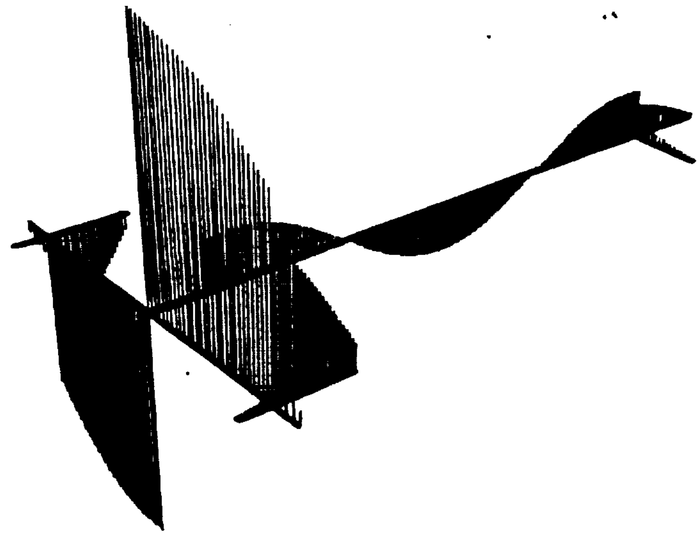

Figure 9. Fourth flow mode of the Engine 1 helium lines downstream of regulators.

Not all regulators oscillated on the vehicle and those that did, oscillated at different frequencies depending on flow demand and number of regulators in use. One mode of oscillation was around 115 to 120 hertz, with the regulators oscillating out of phase with each other. This mode was predicted by ACLMODES and can be seen in Figure 7 and Figure 9.

The pressure mode shapes were also used to determine if the pressure oscillations being measured by a transducer were representative of the oscillations at the regulators. This was done by examining the modes with frequencies near the frequency of interest and determining if the pressure amplitude at the transducer was being attenuated or amplified compared to that of the regulator.

CONCLUSIONS

The method presented herein has proven to be a very useful and accurate tool for determining dynamic characteristics of complex fluid networks, such as pressure recovery and oscillatory behavior. When implemented in a computer code and coupled with a plotting routine, this technique can graphically show vital information about the behavior of a fluid system impossible to obtain with hand calculations.

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APPENDIX A

Modes of a Straight Acoustic Line with General End Conditions

The classical wave equation for a straight tube in terms of volumetric flow rate is

$$c^2 \frac{\partial^2 Q}{\partial x^2} = \frac{\partial^2 Q}{\partial t^2} \tag{A.1}$$

The boundary conditions for the system shown below are derived from the continuity equation and the definition of fluid capacitance.

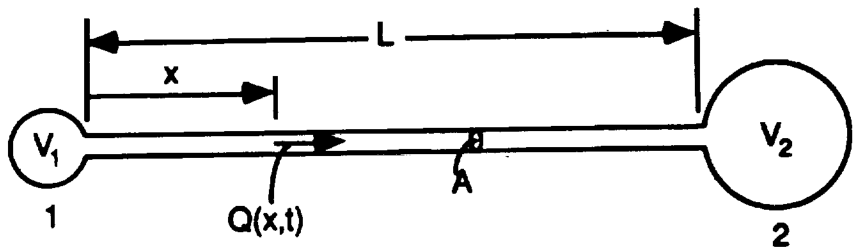

Figure A-1. Straight acoustic line.

The continuity equation is

$$\rho c^2 \frac{\partial Q}{\partial x} = -A \frac{\partial P}{\partial t} \tag{A.2}$$

where

ρ = fluid density
c = acoustic velocity
Q = volumetric flow rate
P = pressure
A = flow area

But

$$\frac{\partial P}{\partial t} = \frac{1}{C}(Q_{in} - Q_{out}) \tag{A.3}$$

where C is the fluid capacitance which is given by

Gas: $$C = \frac{V}{\gamma P} = \frac{V}{\gamma R T} = \frac{V}{\rho c_g^2}$$

Liquid: $$C = \frac{V}{B} = \frac{V}{\rho c_L^2}$$ (B = Bulk Modulus)

Substitution of (A.3) in (A.2) yields

$$\rho c^2 \frac{\partial Q}{\partial x} = -A \frac{1}{C} (Q_{in} - Q_{out})$$

or,

$$\frac{\partial Q}{\partial x} = -\frac{A}{V} (Q_{in} - Q_{out}) \qquad \text{(A.4)}$$

For end 1, x = 0,

$$Q_{in} = 0, \qquad Q_{out} \equiv Q\big|_{x=0}$$

Thus,

$$\frac{\partial Q}{\partial x}\bigg|_{x=0} = \frac{A}{V} Q\big|_{x=0} \qquad \text{(A.5)}$$

Similarly, for end 2, y=L,

$$Q_{in} \equiv Q\big|_{x=L}, \qquad Q_{out} = 0$$

from which

$$\frac{\partial Q}{\partial x}\bigg|_{x=L} = -\frac{A}{V} Q\big|_{x=L} \qquad \text{(A.6)}$$

The general solution of Eq. (A.1) is

$$Q(x,t) = T(t) \left(D_1 \sin \frac{\omega}{c} x + D_2 \cos \frac{\omega}{c} x\right) \qquad \text{(A.7)}$$

where

$$T(t) = B_1 \sin \omega t + B_2 \cos \omega t \qquad \text{(A.8)}$$

Substitution of conditions (A.5) and (A.6) into Eq. (A.7) leads to the following frequency equation

$$\Omega \tan \Omega = \frac{1 + \frac{\alpha_1}{\alpha_2}}{\alpha_1 - \frac{1}{\alpha_2 \Omega^2}} \qquad \text{(A.9)}$$

where $\Omega = \frac{\omega}{c} L$ is a nondimensional frequency, $\alpha_1 = \frac{V_1}{AL}$ and $\alpha_2 = \frac{V_2}{AL}$

Special Cases

1. $V_2 \rightarrow \infty$ (open end)

$$\Omega \tan \Omega = \frac{1}{\alpha_1}$$

2. $V_2 \rightarrow 0$ (closed end)

$$\tan \Omega = -\alpha_1 \Omega$$